Final Examination Solutions
Philosophy 112
Winter 2001

Please work all the problems in the space provided. All problems are weighted eually. You may use only the rule set noted on the individual problems. Please be sure that you do everything that is asked for in each problem.

1. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Show that it is valid in *PD*.

No positive integer is greater than itself; given any positive integer, there is another that is greater. Therefore, there is no greatest positive integer.

UD: The set of all positive integers
Gxy: x is greater than y

$\sim(\exists x)Gxx \ \& \ (\forall x)(\exists y)Gyx$

$\sim(\exists x)(\forall y)Gxy$

1	$\sim(\exists x)Gxx \ \& \ (\forall x)(\exists y)Gyx$	Assumption
2	$\quad (\exists x)(\forall y)Gxy$	Assumption
3	$\quad\quad (\forall y)Gay$	Assumption
4	$\quad\quad Gaa$	3 ∀ E
5	$\quad\quad (\exists x)Gxx$	4 ∃ I
6	$\quad\quad \sim(\exists x)Gxx$	1 & E
7	$\quad\quad \bot$	5 6 ⊥ I
8	$\quad \bot$	8 ⊥ E
9	$\quad \sim(\exists x)(\forall y)Gxy$	2 3-7 ∃ E
10	$\sim(\exists x)(\forall y)Gxy$	2-9 ∼ I

2. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Using any semantical technique for *PL*, determine whether it is quantificationally valid or invalid and defend your answer.

Everything is the same as everything else. So, either everything is good, or nothing is.

UD: Everything
Gx:x is good

$(\forall x)(\forall y)x = y$

$(\forall x)Gx \vee (\forall x){\sim}Gx$

The argument is quantificationally valid. Suppose the premise is true on interpretation I. Then every pair of objects u_1 and u_2 in the UD satisifies the condition specified by '$x = y$'. Therefore, u_1 and u_2 are the same object. Since the choice of u_1 and u_2 was arbitrary, all objects in the UD are identical. Every object in the UD is either in the extension of 'G' or some item is not in the extension of 'G'. If the former condition holds, every object in the UD satisfies 'Gx', in which case the condition specified by '$(\forall x)Gx$' is satisified by every object in the UD, and therefore so is the condition specified by '$(\forall x)Gx \vee (\forall x){\sim}Gx$'. If the latter condition holds, then, since all the objects in the UD are identical, no object in the UD is in the extension of 'G', so that no objects in the UD satisfy the condition specified by 'Gx'. In that case, all objects in the UD satisfy '${\sim}Gx$'. Then all objects in the UD satisfy the condition specified by '$(\forall x){\sim}Gx$', and hence they all satisfy the condition specified by '$(\forall x)Gx \vee$ '$(\forall x){\sim}Gx$', so that it is true. Either way, then, '$(\forall x)Gx \vee (\forall x){\sim}Gx$' is true on I, and so the argument is quantificationally valid.

3. Using the definitions from the formal semantics, show that the following two sentences are quantificationally equivalent.

(∃x)Gx
∼(∀x)∼Gx.

Let **I** be an arbitrary interpretation. '(∃x)Gx' is true on **I** if and only if it is satisfied by all variable-assignments **d** for **I**. This holds if and only if 'Gx' is satisfied by some x-variant **d**[**u**/x] of all variable-assignments **d**. And this holds if and only if '∼Gx' is not satisfied by some x-variant **d**[**u**/x] of all variable-assignments **d**. This holds if and only if '(∀x)∼Gx' is not satisifed by all variable assignments **d**. And this holds if and only if '∼(∀x)∼Gx' is satisified by all variable assignments **d**, which in turn holds if and only if the sentence is true on **I**. Therefore, '(∃x)Gx' is true on **I** if and only if '∼(∀x)∼Gx' is true on **I**, for arbitrary **I**, which was to be proved.

4. Show that the following sentence is quantificationally indeterminate by constructing an interpretation on which it is true and one on which it is false. State why it is true and false on the two interpretations, respectively.

(∃x)(∀y)(∀z)(x=z ∨ y=z))

For an interprtation **I** on which the sentence is true, let the UD of **I** consist of a single object, the number 1. As with any object, 1 is identical with itself. Also, there is only one variable-assignment **d**, since each variable can be assigned only the number 1. Thus **d** satisfies 'x=z', and so it satisifies 'x=z ∨ y=z'. Since (trivially) this holds for all z-variants of **d**, **d** satisifies '(∀z)(x=z ∨ y=z)'. Similarly, **d** satisfies '(∀y)(∀z)(x=z ∨ y=z)', since all its y-variants satisfy the sub-formula. And so there is an x-variant of **d** (i.e., **d** itself) which satisfies '(∀y)(∀z)(x=z ∨ y=z)', in which case **d** satisifies '(∃x)(∀y)(∀z)(x=z ∨ y=z)'. Then the sentence is true on **I**.

For an interpretation on which the sentence is false, we must use a domain with at least three objects. Let the UD = {1, 2, 3}. For any variable-assignment **d**, **d**(x)=1, **d**(x)=2, or **d**(x)=3. Now suppose **d**(x)=1. **d**[1/x, 2/y, 3/z] does not satisfy 'x = z' or 'y = z'. Therefore, it does not satisfy 'x = z ∨ y = z'. So not all z-variants of **d**[1/x, 2/y] satisfy 'x = z ∨ y = z', in which case **d**[1/x, 2/y] does not satisfy '(∀z)(x=z ∨ y=z)'. So not all y-variants of **d**[1/x] satisfy '(∀z)(x=z ∨ y=z)', in which case **d**[1/x] does not satisfy '(∀y)(∀z)(x=z ∨ y=z)'. If **d**(x)=2, the same reasoning applies for **d**[2/x, 1/y, 3/z]. And if **d**(x)=3, this reasoning applies for **d**[3/x, 1/y, 2/z]. So there is no x-variant of **d** which satisfies '(∀y)(∀z)(x=z ∨ y=z)'. So **d** does not satisfy '(∃x)(∀y)(∀z)(x=z ∨ y=z)', and the sentence is false on **I**, which was to be proved.

5. Show that the following set of sentences is quantificationally consistent by constructing an appropriate expanded truth-table.

$\{(\forall y)(\exists x){\sim}x{=}y,\ (\exists z)Faz,\ (\exists z)Fza\}$

N/A

6. Using the formal semantics for *PL*, determine the truth-value of the following sentence on the interpretation given. Show in detail how the truth-value is determined.

(∀x)(Ex ⊃ (∃y)Lyx)

UD: {1,2}
E: {<**u**>: **u** is even}
L: $\{<\mathbf{u}_1,\mathbf{u}_2>$: $\mathbf{u}_1$ is less than $\mathbf{u}_2\}$

This sentence is true on the given interpretation.
Let **d** be a variable-assignment for the interpretation. Then **d**(x) = 1 or **d**(x) = 2. If **d**(x) = 1, then **d** does not satisfy the antecedent of the conditional, 'Ex', since 1 is not in **I**(E). Therefore, **d** satisfies the conditional 'Ex ⊃ (∃y)Lyx'. If **d**(x)=2, then **d**[1/y] satisfies 'Lyx',' since the pair <1, 2> is in **I**(L). So, **d** satisfies '(∃y)Lxy', and hence it satisfies 'Ex ⊃ (∃y)Lyx'. So all of the (two) x-variants of **d** satisfy '(Ex ⊃ (∃y)Lyx)', in which case the sentence '(∀x)(Ex ⊃ (∃y)Lyx)' is satisified by **d**. Since the choice of **d** is arbitrary, it is satisified by all variable-assignments, in which case the original sentence is true on **I**.

(This problem did not make it to the printed final.)

Prove that the following derivability relation holds in *PD*.

$\{(\forall x)(\exists y)[Ixy \,\&\, (\forall z)(Ixz \supset z{=}x)], (\forall x)(\forall y)(Iyx \equiv Fyx)\} \vdash (\forall x)(\exists y)[Fyx \,\&\, (\forall z)(Fzy \supset z{=}y)]$

1	$(\forall x)(\exists y)[Ixy \,\&\, (\forall z)(Ix \supset z{=}x)]$	Assumption
2	$(\forall x)(\forall y)(Iyx \equiv Fyx)$	Assumption
3	$(\exists y)[Iay \,\&\, (\forall z)(Iaz \supset z{=}a)]$	1 $\forall$ E
4	$Iab \,\&\, (\forall z)(Iaz \supset z{=}a)$	Assumption
5	Iab	4 & E
6	$(\forall y)(Iyb \equiv Fyb)$	2 $\forall$ E
7	$Iab \equiv Fab$	6 $\forall$ E
8	Fab	5 7 $\equiv$ E
9	Fab	Assumption
10	$(\forall z)(Iaz \supset z = a)$	4 & E
11	$Iab \supset b = a$	10 $\forall$ E
12	$b = a$	5 11 $\supset$ E
13	$a = a$	12 = E
14	$a = b$	12 13 = E
15	$Fab \supset a = b$	9-14 $\supset$ I
16	$(\forall z)(Fzb \supset z = b)$	15 $\forall$ I
17	$Fab \,\&\, (\forall z)(Fzb \supset z = b)$	8 16 & I
18	$(\exists y)[Fay \,\&\, (\forall z)(Fzy \supset z = y)]$	17 $\exists$ I
19	$(\forall x)(\exists y)[Fxy \,\&\, (\forall z)(Fzy \supset z = y)]$	18 $\forall$ I
20	$(\forall x)(\exists y)[Fxy \,\&\, (\forall z)(Fzy \supset z = y)]$	3 4-19 $\exists$ E