Solutions to Philosophy 112 Second Midterm
Winter, 2001

1. Prove that the following derivability relation holds in *PD*.

{(∃x)(∀y)[(Ay & By) ⊃ Cxy], (∀y)(Ay ⊃ By)} ⊢ (∀y)(Ay ⊃ (∃x)Cxy)

1	(∃x)(∀y)[(Ay & By) ⊃ Cxy]	Assumption
2	(∀y)(Ay ⊃ By)	Assumption
3	(∀y)[(Ay & By) ⊃ Cay]	Assumption
4	Ac	Assumption
5	Ac ⊃ Bc	2 ∀E
6	Bc	4 5 ⊃E
7	Ac &Bc	4 6 & I
8	(Ac & Bc) ⊃ Cac	3 ∀ E
9	Cac	7 8 ⊃ E
10	(∃x)Cxc	9 ∃ I
11	Ac ⊃ (∃x)Cxc	4-10 ⊃ I
12	(∀y)(Ay ⊃ (∃x)Cxy)	11 ∀ I
13	(∀y)(Ay ⊃ (∃x)Cxy)	1 3-12 ∃ E

2. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Show that it is valid in *PD*.

Someone took something from the shelf. Anyone who took anything from the shelf was in the room last night. Therefore, someone was in the room last night.

UD: Everything
s: the shelf — r: the room
Px: x is a person — Ixy: x was in y last night
Txyz: x took y from z

1	(∃x)(Px & (∃y)Txys)	Assumption
2	(∀x)[Px ⊃ ((∃y)Txys ⊃ Ixr)]	Assumption
3	Pa & (∃y)Tays	Assumption
4	Pa	3 & E
5	(∃y)Tays	3 & E
6	Tabs	Assumption
7	Pa ⊃ ((∃y)Tays ⊃ Iar))	2 ∀ E
8	(∃y)Tays ⊃ Iar	4 7 ⊃ E
9	Iar	5 8 ⊃ E
10	Pa & Iar	4 9 & I
11	(∃x)(Px & Ixr)	10 ∃ I
12	(∃x)(Px & Ixr)	5 6-11 ∃ E
13	(∃x)(Px & Ixr)	1 3-12 ∃ E

3. Prove the equivalence of the following two sentences in *PD+*.

$(\forall x)(Ax \supset Bx)$, $\sim(\exists x)(Ax \,\&\, \sim Bx)$

1	$(\forall x)(Ax \supset Bx)$	Assumption
2	$(\forall x)(\sim Ax \vee Bx)$	1 Impl
3	$(\forall x)(\sim Ax \vee \sim\sim Bx)$	2 DN
4	$(\forall x)\sim(Ax \,\&\, \sim Bx)$	3 DeM
5	$\sim(\exists x)(Ax \,\&\, \sim Bx)$	4 QN

1	$\sim(\exists x)(Ax \,\&\, \sim Bx)$	Assumption
2	$(\forall x)\sim(Ax \,\&\, \sim Bx)$	1 QN
3	$(\forall x)(\sim Ax \vee \sim\sim Bx)$	2 DeM
4	$(\forall x)(\sim Ax \vee Bx)$	3 DN
5	$(\forall x)(Ax \supset Bx)$	4 Impl

4. Prove that the following is a theorem of *PDI+*.

$Fa \equiv (\exists y)(y = a \,\&\, Fy)$

1	Fa	Assumption
2	$(\forall x)x = x$	$=$ I
3	$a = a$	1 $\forall$ E
4	$a = a \,\&\, Fa$	1 3 & I
5	$(\exists y)(y = a \,\&\, Fa)$	4 $\exists$ I
6	$(\exists y)(y = a \,\&\, Fa)$	Assumption
7	$b = a \,\&\, Fa$	Assumption
8	Fa	7 & E
9	Fa	6 7-8 $\exists$ E
10	$Fa \equiv (\exists y)(y = a \,\&\, Fy)$	1-5 6-9 $\equiv$ I

5. Prove that the following set of sentences is inconsistent in *PD*.

{(∀x)(∀y)(Fxy ⊃Fyx), (∀x)(∀y)(Fxy ⊃ ∼Fyx), (∃x)(∃y)Fxy}

1	(∀x)(∀y)(Fxy ⊃Fyx)	Assumption
2	(∀x)(∀y)(Fxy ⊃ ∼Fyx)	Assumption
3	(∃x)(∃y)Fxy	Assumption
4	(∃y)Fay	Assumption
5	Fab	Assumption
6	(∀y)(Fay ⊃ Fya)	1 ∀ E
7	Fab ⊃ Fba	6 ∀ E
8	Fba	5 7 ⊃ E
9	(∀y)(Fay ⊃ ∼Fya)	2 ∀ E
10	Fab ⊃ ∼Fba	9 ∀ E
11	∼Fba	8 10 ⊃ E
12	⊥	8 11 ⊥ I
13	⊥	4 5-12 ∃ E
14	⊥	3 4-13
15	∼(∃x)(∃y)Fxy	14 ⊥ E