Solutions to Philosophy 112 Second Midterm
Winter, 2002

1. Prove that the following is a theorem of *PD*. This is a symbolization of the barber paradox: no barber shaves all and only those who do not shave themselves.

∼(∃x)(∀y)(Sxy ≡ ∼Syy)

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1    | (∃x)(∀y)(Sxy ≡ ∼Syy)        Assumption
     |---------------------
2    | | (∀y)(Say ≡ ∼Syy)          Assumption
     | |-----------------
3    | | Saa ≡ ∼Saa                2 ∀ E
4    | | | Saa                     Assumption
     | | |----
5    | | | ∼Saa                    3 4 ≡ E
6    | | ∼Saa                      4-5 ∼ I
7    | | Saa                       3 6 ≡ E
8    | | ⊥                         6 7 ⊥ I
9    | ⊥                           1 2-8 ∃ E
10   | ∼(∃x)(∀y)(Sxy ≡ ∼Syy)       9 ⊥ E
11   ∼(∃x)(∀y)(Sxy ≡ ∼Syy)         1-10 ∼ I
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2. Prove the equivalence of the following two sentences of *PD*.

(∀x)(∀y)(Fx ⊃ Gy)
(∀x)(Fx ⊃ (∀y)Gy)

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1    | (∀x)(∀y)(Fx ⊃ Gy)     Assumption
     |------------------
2    | (∀y)(Fa ⊃ Gy)         1 ∀ E
3    | Fa ⊃ Gb               2 ∀ E
4    | | Fa                  Assumption
     | |---
5    | | Gb                  3 4 ⊃ E
6    | | (∀y)Gy              5 ∀ I
7    | Fa ⊃ (∀y)Gy           4-6 ⊃ I
8    | (∀x)(Fx ⊃ (∀y)Gy)     7 ∀ I
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1    | (∀x)(Fx ⊃ (∀y)Gy)     Assumption
     |------------------
2    | Fa ⊃ (∀y)Gy           1 ∀ E
3    | | Fa                  Assumption
     | |---
4    | | (∀y)Gy              2 3 ⊃ E
5    | | Gb                  4 ∀ E
6    | Fa ⊃ Gb               1 ∀ E
7    | (∀y)(Fa ⊃ Gy)         6 ∀ I
8    | (∀x)(∀y)(Fx ⊃ Gy)     7 ∀ I
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3. Symbolize the following argument, revealing as much structure as possible and providing a symbolization key. Show that it is valid in *PD+*.

No number is smaller than itself. Hence, there is no number that is smaller than all numbers.

UD: Numbers
Sxy: x is smaller than y

1	$\sim(\exists x)Sxx$	Assumption
2	$(\forall x)\sim Sxx$	1 QN
3	$\sim Saa$	1 $\forall$ E
4	$(\exists y)\sim Say$	3 $\exists$ I
5	$(\forall x)(\exists y)\sim Sxy$	4 $\forall$ I
6	$(\forall x)\sim(\forall y)Sxy$	5 QN
7	$\sim(\exists x)(\forall y)Sxy$	6 QN

4. Show that the following argument is valid in *PD+*. (This symbolizes an argument from Plato's *Meno*. Something can be taught only if there are teachers and pupils of it. Virtue has no teachers or pupils, so no virtue can be taught.)

$(\forall x)(Cx \supset ((\exists y)Tyx \;\&\; (\exists y)Pyx))$
$(\forall x)(Vx \supset \sim(\exists y)(Tyx \vee Pyx))$

$\sim(\exists x)(Vx \;\&\; Cx)$

1	$(\forall x)(Cx \supset ((\exists y)Tyx \;\&\; (\exists y)Pyx))$	Assumption
2	$(\forall x)(Vx \supset \sim(\exists y)(Tyx \vee Pyx))$	Assumption
3	$(\exists x)(Vx \;\&\; Cx)$	Assumption
4	$Va \;\&\; Ca$	Assumption
5	Va	4 & E
6	Ca	4 & E
7	$Ca \supset ((\exists y)Tya \;\&\; (\exists y)Pya)$	1 $\forall$ E
8	$Va \supset \sim(\exists y)(Tya \vee Pya)$	2 $\forall$ E
9	$(\exists y)Tya \;\&\; (\exists y)Pya$	6 7 $\supset$ E
10	$\sim(\exists y)(Tya \vee Pya)$	5 8 $\supset$ E
11	$(\forall y)\sim(Tya \vee Pya)$	10 QN
12	$\sim(Tba \vee Pba)$	11 $\forall$ E
13	$\sim Tba \;\&\; \sim Pba$	12 DeM
14	$\sim Tba$	13 & E
15	$(\forall x)\sim Txa$	14 $\forall$ I
16	$\sim(\exists y)Tya$	15 QN
17	$(\exists y)Tya$	9 & E
18	$\perp$	16 17$\perp$ I
19	$\perp$	3 4-18 $\exists$ E
20	$\sim(\exists x)(Vx \;\&\; Cx)$	19 $\perp$ E
21	$\sim(\exists x)(Vx \;\&\; Cx)$	3-20 $\sim$ I

5. Prove that the following set of sentences is inconsistent in *PDI*.

$\{(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y)), \sim g = a, Fa, Fg\}$

1	$(\exists x)(Fx \ \& \ (\forall y)(Fy \supset x = y))$	Assumption
2	$\sim g = a$	Assumption
3	Fa	Assumption
4	Fg	Assumption
5	$Fc \ \& \ (\forall y)(Fy \supset c = y)$	Assumption
6	$(\forall y)(Fy \supset c = y)$	5 & E
7	$Fa \supset c = a$	6 $\forall$ E
8	$c = a$	3 7 $\supset$ E
9	$\sim g = c$	2 8 = E
10	$Fg \supset c = g$	6 $\forall$ E
11	$c = g$	4 10 $\supset$ E
12	$g = a$	8 11 = E
13	$g = a$	1 5-12 $\exists$ E