

## Object Language and Metalanguage

### Formal Languages

- Sentence Logic and Predicate Logic are **formal languages**.
- A formal language is a set of **sentences** generated by **rules of formation** from a **vocabulary**.
- The sentences of Sentence Logic and Predicate Logic are not part of natural language (though some may resemble natural-language sentences).
- The formal languages Sentence Logic and Predicate Logic are the objects of our study, and as such they are called **object languages**.

### The Metalanguage

- If we are going to state anything about an object language, we must make use of a language.
- We call a language used to study an object language a **metalanguage**.
- In theory, the metalanguage may be identical to or include the object language.
  - We use English to study English in linguistics.
- We will strictly separate our metalanguage (English with some extra technical vocabulary) from our object languages.
- Keeping the languages separate allows us to avoid semantical paradox (Tarski).

### Use and Mention

- When we talk about an item of language, we are said to **mention** it.
- Whenever an item of any object language is mentioned, it must be placed within single quotation marks.
- We may **use** English to mention an item of English.
  - ‘Bush’ has four letters and starts with a ‘B’.
  - ‘George W. Bush was born in Texas’ is false.
  - ‘This sentence is false’ is true.

### Metavariables

- We may also use English to mention items of Sentence Logic and Predicate Logic.
  - ‘ $\supset$ ’ is a connective of Sentence Logic.
  - ‘ $P \supset Q$ ’ is a conditional.
  - If ‘ $P$ ’ is true and ‘ $P \supset Q$ ’ is true, then ‘ $Q$ ’ is true.
- To state general facts about Sentence Logic and Predicate Logic, we must use expressions that designate classes of items of the object language.
- Such expressions are called **metavariables**.

### Metavariables for Sentences and Sets of Sentences

- To mention metavariables themselves, single quotation marks must be used.
- ‘ $Q$ ’ through ‘ $Z$ ’ will be used as metavariables for sentences of both Sentence Logic and Predicate Logic.
- ‘ $X$ ’ through ‘ $Z$ ’ will be used as metavariables for sets of sentences of both Sentence Logic and Predicate Logic.

### Metavariables and Connectives

- In English, we refer to connectives of Sentence Logic using such expressions as ‘sign of negation’ or ‘sign of the conditional’.
- We may also refer to them by mentioning them:
  - ‘ $\sim$ ’
  - ‘ $\supset$ ’
- We must have a way to combine our use of metavariables with reference to connectives.

### Names of Themselves

- We could make general statements about connectives using English.
  - If  $X$  is a sentence of Sentence Logic, then the result of prefixing the sentence  $X$  refers to with a ‘ $\sim$ ’ (or sign of negation) and surrounding the result with parentheses is a sentence of Sentence Logic.
- This kind of statement is obviously very cumbersome.
- We want to say: ‘If  $X$  is a sentence, then  $(\sim X)$  is a sentence’.
- Strictly speaking, this mixes the object language with the metalanguage.
- So we say (without paradox) that in ‘ $\sim X$ ’, ‘ $\sim$ ’ is used as a name of itself, or autonomously (Carnap).

### Metavariables for Predicate Logic

- The vocabulary of Predicate Logic is an extension of the vocabulary of Sentence Logic.
- It contains all the expressions of the vocabulary of Sentence Logic along with other expressions proper to Predicate Logic.
- The metavariables for these new vocabulary items will be introduced when the vocabulary items themselves are introduced.

### Sets

- An important part of the metalanguage for Sentence Logic and Predicate Logic is set-theoretic notation.
  - We have already described metavariables referring to sets of sentences.
- Sets are indicated by enclosure in curly brackets '{' and '}'.
- '{P, P  $\supset$  Q}' indicates the set consisting of 'P' and 'P  $\supset$  Q'.
- Ordered sets are indicated by enclosure in angle brackets '<' and '>'. '<P, P  $\supset$  Q>' indicates the set consisting of 'P' and 'P  $\supset$  Q' in that order: first 'P' and then 'P  $\supset$  Q'.

### Specifying Sets

- There are two methods of specifying the contents of a set.
- The method of **enumeration** simply lists the contents of the set using linguistic items that refer to the objects in the set.
  - {The President of the United States, Donald Rumsfeld, 2005}
  - <Adam, Eve>

The method of **description** states what condition must be satisfied for something to be a member of the set.

- {x: x is blond}: the set of all x such that x is blond, or the set of all blonds
- {(x,y): x > y}: the set of all ordered pairs x and y such that x is greater than y