Solutions to Selected Exercises Using Formal Semantics

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2-5 a6)
To solve this problem, we must appeal to our general knowledge. I know of at least one unhappy U.S. citizen over 21, Kyle, who is not a millionaire. Let d be an arbitrary variable assignment. $\langle$Kyle$\rangle$ $\notin$ v(M), and $\langle$Kyle$\rangle$ $\notin$ v(H), so $\langle$d[Kyle/x](x)$\rangle$ is not in either v(M) or v(H). So, d[Kyle/x] does not satisfy 'Hx' or 'Mx'. So d[Kyle/x] satisifies '∼Hx' and '∼Mx'. Therefore, d[Kyle/x] satisifies '∼Hx & ∼Mx'. Thus, d[Kyle/x] satisfies '(Hx & Mx) ∨ (∼Hx & ∼Mx)'. So d satisfies '(∃x)[(Hx & Mx) ∨ (∼Hx & ∼Mx)]. Since d is arbitrary, the sentence is satisified by all variable assignments, and the sentence is true in the interpretation.

2-5 a8)
I know a number of U.S. citizens over 21 who are happy but not millionaires. Josh is one of them. $\langle$Josh$\rangle$ $\notin$ v(M), so for arbitrary variable assignment d, $\langle$d[Josh/x](x)$\rangle$ $\notin$ v(M), in which case d[Josh/x] does not satisfy 'Mx'. But $\langle$Josh$\rangle$ $\in$ v(H), and so $\langle$d[Josh/x](x)$\rangle$ $\in$ v(H) and d[Josh/x] satisfies 'Hx'. Then d[Josh/x] does not satisfy 'Hx ⊃ Mx'. Therefore, d does not satisfy '(∀x)(Hx ⊃ Mx)'. It follows that d satsifies '(∀x)(Hx ⊃ Mx) ⊃ ∼(∃x)Mx'. Since d is arbitrary, all variable assignments satisfy the sentence, and the sentence is true in the given interpretation.

2-5 b7)
The number 5 is odd but is not greater than or equal to 17. Since $\langle 5\rangle \in$ v(O), and hence $\langle$d[5/x](x) $\in$ v(O), d[5/x] satisfies 'Ox'. Further, since $\langle 5,18\rangle \notin$ v(K), so neither is $\langle$d[5/x](x),v(a_{17})$\rangle$ a member of v(K). Hence d[5/x] does not satisfy 'Kxa_{17}', in which case it does not satisfy '∼Kxa_{18} & Kxa_{17}'. Therefore, d[5/x] does not satisfy 'Ox ≡ (∼Kxa_{18} & Kxa_{17})'. Since there is at least one x-variant of d whose value is a member of the domain that does not satisfy the open sentence, the universally quantified sentence '(∀x)[Ox ≡ (∼Kxa_{18} & Kxa_{17})]' is false in the interpretation.

2-6 d)

Let **I** be an interpretation which makes '(∀x)(Bx & Lxe)' true. Then for all variable assignments **d** based on **I**, the sentence is satisfied. This holds just in case for all members **u** of the domain of **I**, **d**[**u**/x] satisfies 'Bx & Lxe'. Thus, **d**[**u**/x] satisfies both 'Bx' and 'Lxe', in which case it satisfies 'Bx'. Then **d** satisfies '(∀x)Bx', and since this holds for all variable assignments based on **I**, the sentence is true in **I**. So given that the premise is true in an interpretation, so is the conclusion, and the argument is valid.