Natural Deduction Rules with Iterated Quantifiers

Iterated Universal Quantifiers

- There are no restrictions on instantiating universal sentences, which means there is no problem with iterated universal quantifiers.
- For example, '$(\forall x)(\forall y)Fxy$' can be instantiated as:
  - $(\forall y)Fay$, which can be instantiated as:
    - Faa, or
    - Fab, etc.

Iterated Existential Quantifiers

- The instantiation of existential quantifiers is subject to restrictions which rule out some instantiations in the case of multiple quantifiers.
- For example, the following pattern of instantiation is *not* permitted:

1		$(\exists x)(\exists y)Fxy$	P
2	a	$(\exists y)Fay$	A
3	a	Faa	A
4			

- The second instantiation violates the restriction that the instantiating name be isolated in the derivation.

Hard Problems

- On page 94, a derivation in 21 steps is given.
- We are told what the two main strategic moves are, but not why they were chosen.
- Reductio is chosen as the basic strategy.
  - The reason is that it would be impossible to use $\exists$ I to get the conclusion.
  - This condition usually holds, and so reductio is a good strategy for deriving existential sentences.
- The sub-strategy is to use reductio again to get '$P\hat{a}$' in order to use $\forall$ I.
  - Once again, it is impossible to get this result in any other way.
  - This condition usually holds, so reductio is a good strategy for deriving atomic sentences that cannot be derived using instantiation.