

Philosophy 134
Spring, 2007
Homework 3

Due: April 25, 2007, in class

1. Suppose we were to introduce the following two rules of inference for the ‘ \diamond .’

\diamond Elimination

$$\left| \begin{array}{l} \diamond\alpha \quad \text{Already Derived} \\ \vdots \\ \alpha \end{array} \right| \quad \diamond E$$

Provided that α is written to the right of exactly one \diamond -restricted scope line.

\diamond Introduction

$$\left| \begin{array}{l} \vdots \\ \alpha \end{array} \right| \quad \diamond I$$

Provided that α is not to the right of any other scope line.

What other proviso would need to be added to \diamond Elimination to preserve soundness of the two rules relative to the basic modal semantics? (Try to think of an invalid inference that could be made given the single stated proviso, using *both rules*.)

2. Give a **semantical** proof that $\diamond\sim\alpha$ is **semantically** equivalent to $\sim\Box\alpha$.
3. Give two **derivations** to prove that $\diamond\sim\alpha$ is **derivationally** equivalent to $\sim\Box\alpha$.
4. Propose and defend a rule of Strict Reiteration for the ‘ \forall ’ and of \forall Introduction.
5. Derive the following theorem in the basic derivational system: $(\Box A \supset \diamond B) \supset \diamond(A \supset B)$.