

Philosophy 134
Exercises

The following is a list of sentences which are valid in (and therefore theorems of) various systems of modal logic studied in the course. For each sentence associated with the system, you may attempt to give semantical arguments or derivations to show that the sentence is, respectively, valid in or derivable in that system. For any sentence in a system stronger than K , you may give a counter-example which shows that it is not valid in a weaker system.

System K

1. $\Box(P \& Q) \supset (\Box P \& \Box Q)$
2. $\Diamond(P \& Q) \supset \Diamond P$
3. $\sim \Diamond P \supset \Box(P \supset Q)$
4. $\Box P \supset (\Diamond Q \supset \Diamond(P \& Q))$
5. $\Diamond(P \supset Q) \supset (\Box P \supset \Diamond Q)$
6. $((P \prec Q) \& (Q \prec R)) \supset (P \prec R)$
7. $((P \prec Q) \& \Diamond(P \& R)) \supset \Diamond(Q \& R)$
8. $((P \prec R) \& (Q \prec S)) \& (\Box P \vee \Box Q) \prec \Box(R \vee S)$ (corrected)
9. $\Diamond(P \supset Q) \vee \Box(Q \supset P)$

System D (contains K)

1. $\Diamond Q \vee \Diamond \sim Q$
2. $\sim (\Box P \& \Box \sim P)$

System T (contains D)

1. $\Diamond \sim P \vee (\Diamond \sim Q \vee \Diamond(P \& Q))$
2. $\Box(P \supset (Q \& R)) \supset ((\Box P \supset \Diamond P) \& (\Box P \supset \Diamond R))$

System S_4 (contains T)

1. $\Diamond \Box P \prec \Diamond P$
2. $\Box(P \supset (Q \supset R)) \prec \Box(\Box P \supset (\Box Q \supset \Box R))$
3. $(\Box P \vee \Box Q) \equiv \Box(\Box P \vee \Box Q)$
4. $(\Box(P \equiv Q) \prec R) \prec (\Box(P \equiv Q) \prec \Box R)$
5. $(P \prec Q) \prec (\Box P \prec \Box Q)$

System B (contains T)

1. $\Box(P \supset \Box Q) \supset (\Diamond P \supset Q)$
2. $\Box(\Diamond P \supset Q) \supset (P \supset \Box Q)$
3. $\Box \Box(P \supset \Box \Diamond P)$
4. $\Box P \supset \Box \Box \Diamond P$

System S_5 (contains S_4 and B)

1. $\Box(\Box P \vee Q) \supset (\Box P \vee \Box Q)$
2. $(\Diamond P \& \Diamond Q) \supset \Diamond(\Diamond P \& Q)$
3. $\Diamond(P \prec Q) \supset (\Box P \supset \Box Q)$
4. $(\Box P \supset \Box Q) \supset (\Box P \prec \Box Q)$

