Descartes's Model of Reflection and Refraction

Philosophy 168
G. J. Mattey

Reflection and refraction

- In his 1637 *Optics,* Descartes offered an explanation of the phenomena of reflection and refraction
- The goal is to generate rules which predict accurately the behavior of light when reflected or refracted
- The explanation is carried out in geometrical terms
- It relies on several crucial assumptions

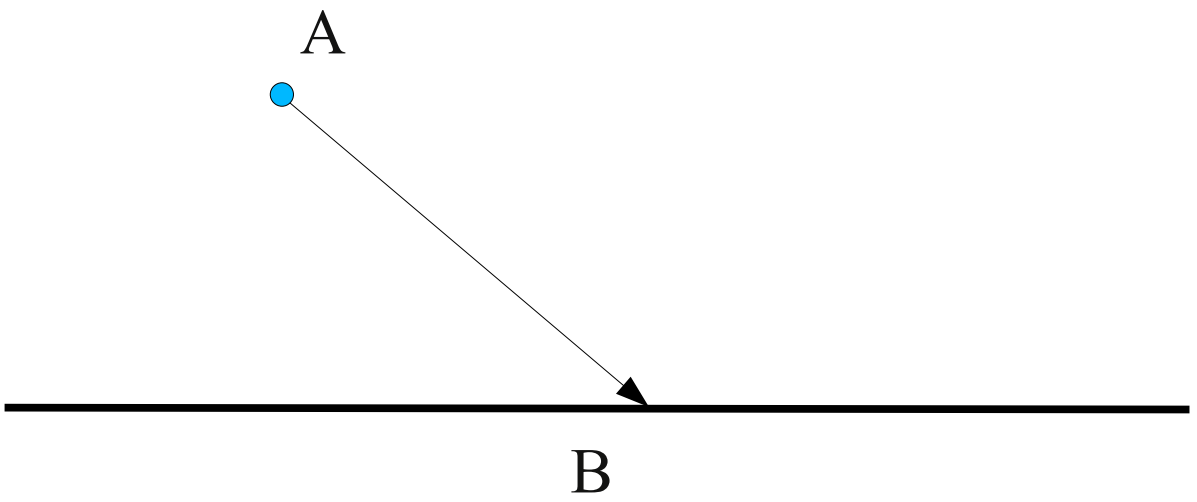
Theoretical assumptions

- The explanation of both reflection and refraction uses as a model the behaviour of a tennis ball hit toward a surface by a racquet
- Three theoretical assumptions are made
  - The determinant of motion and the determinant of the direction of motion are distinguished
  - The motion of the ball can be decomposed into a horizontal and a vertical component
  - The motion of the ball can be represented geometrically with straight lines

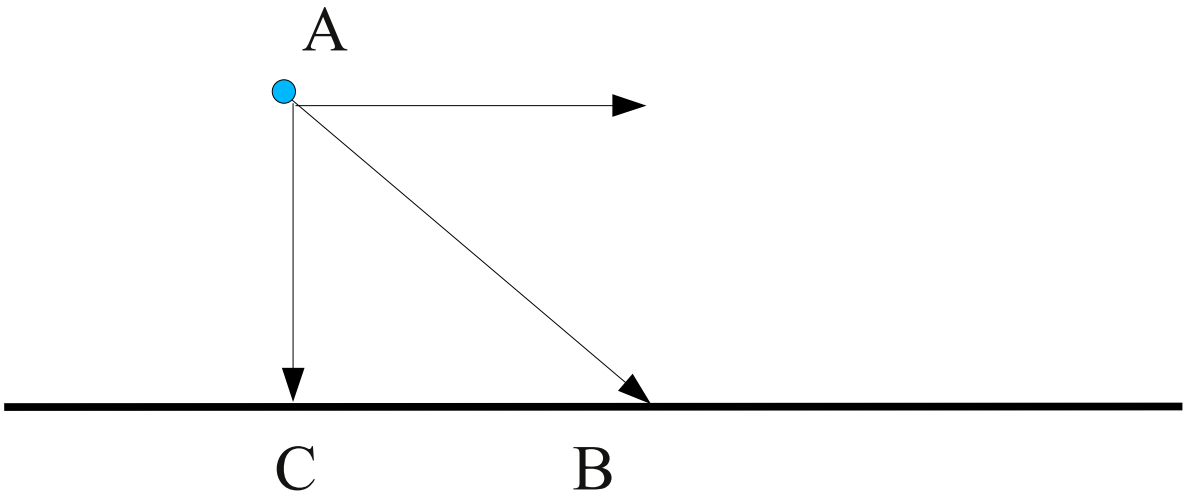
Idealizing assumptions for reflection

- The ball moves at a constant speed through its whole path
- The ball moves toward the ground, which is perfectly flat and hard
- The size, shape, weight of ball have no effect on the motion of the ball

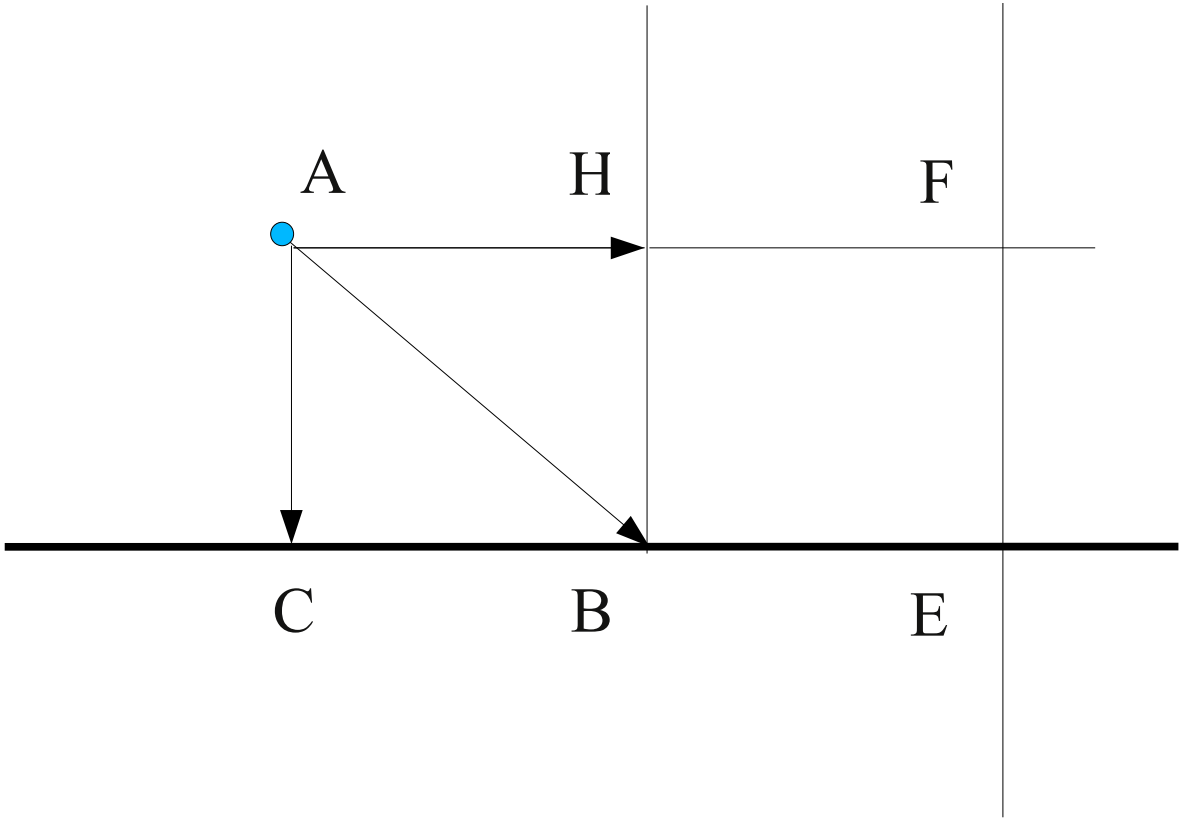
Representation of initial motion



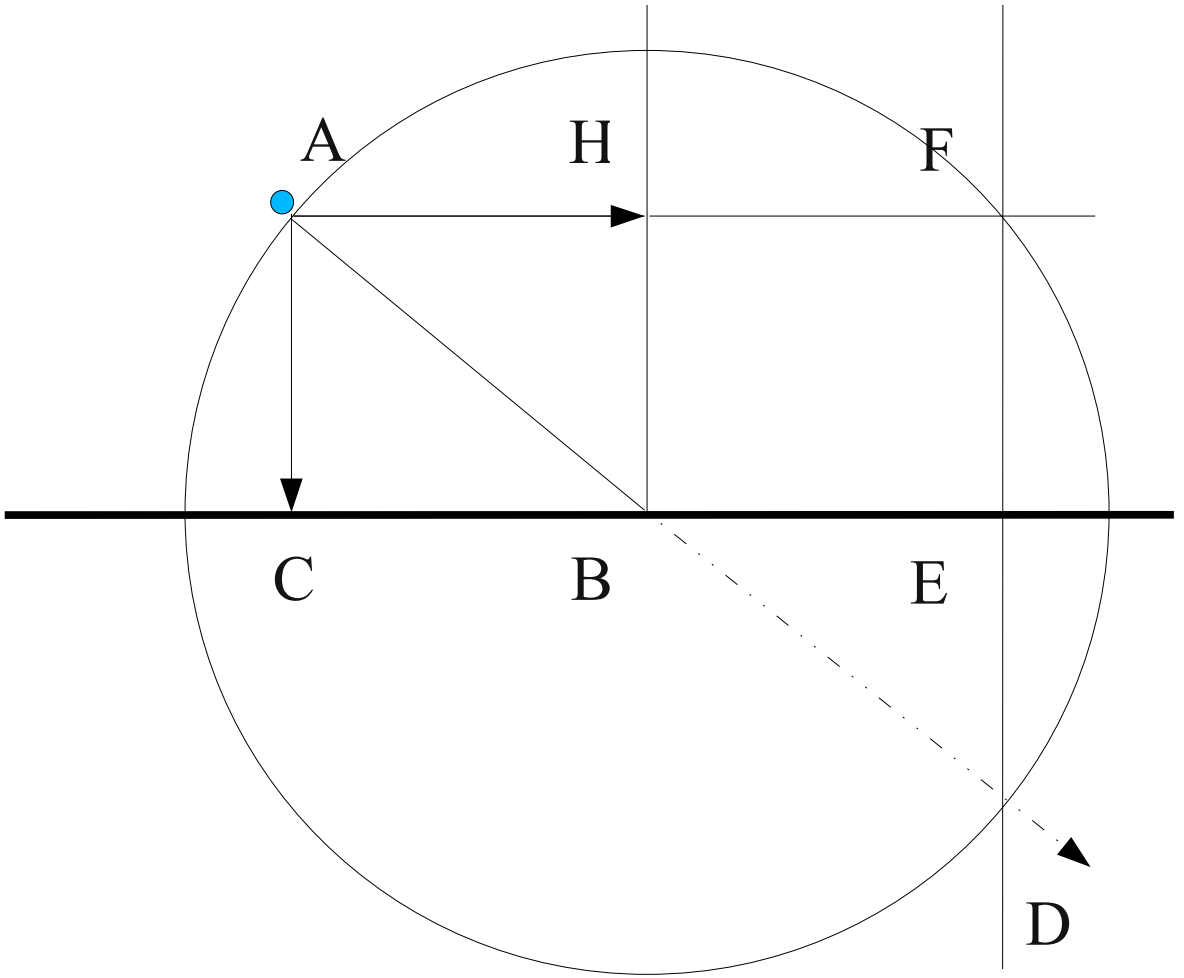
Decomposition of horizontal and vertical components



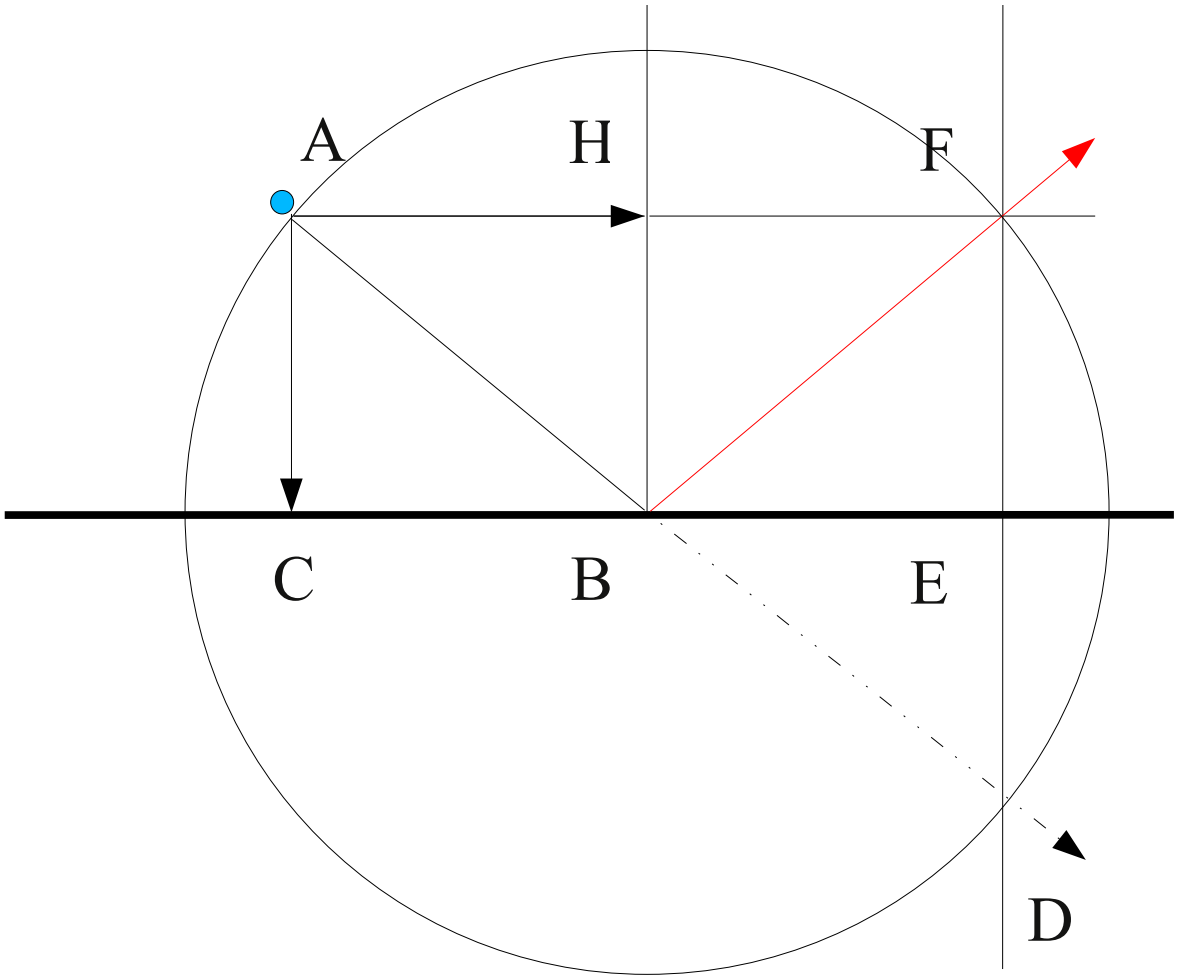
Duplication of the horizontal component



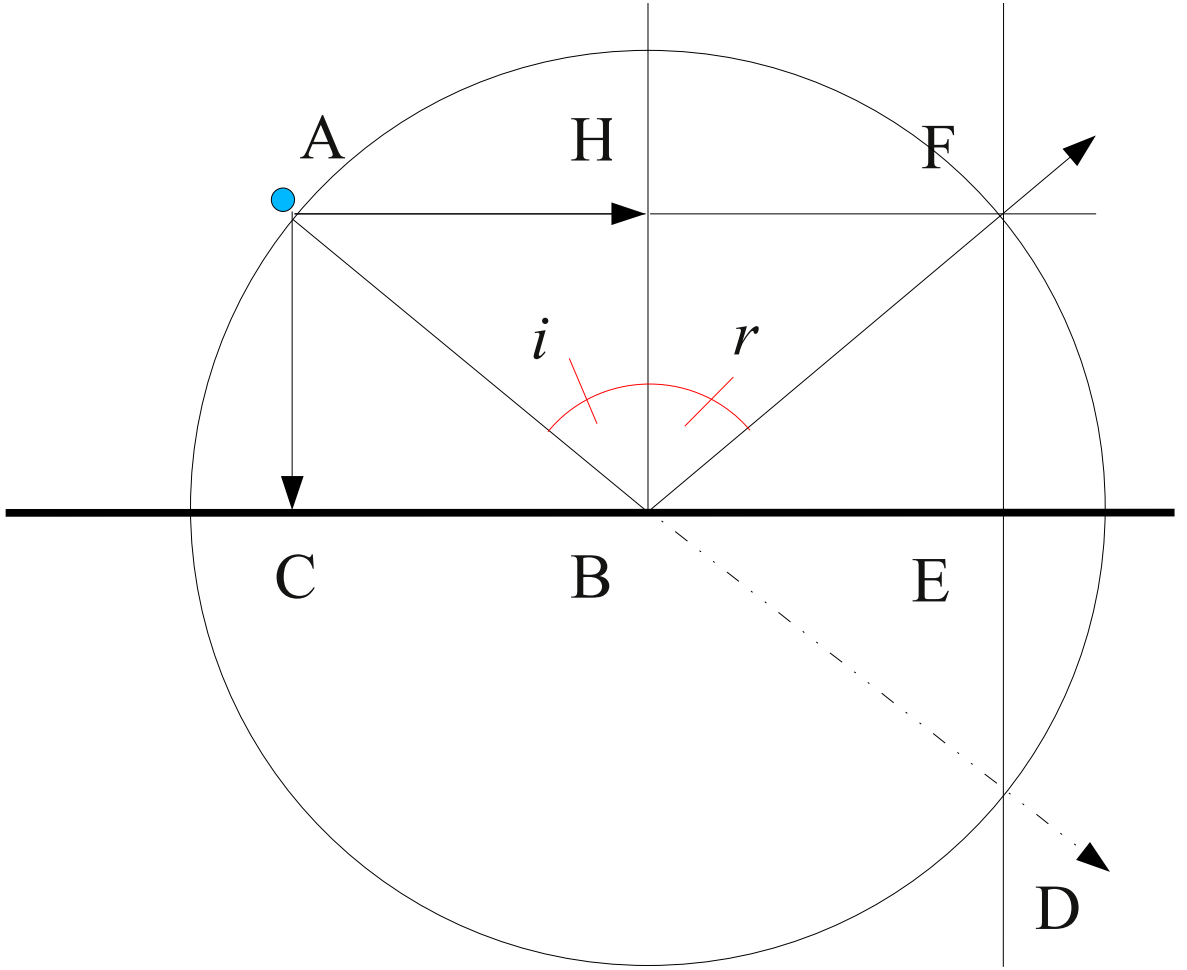
Duplication of the distance travelled



Determination of the path



Determination of the angles of incidence and reflection



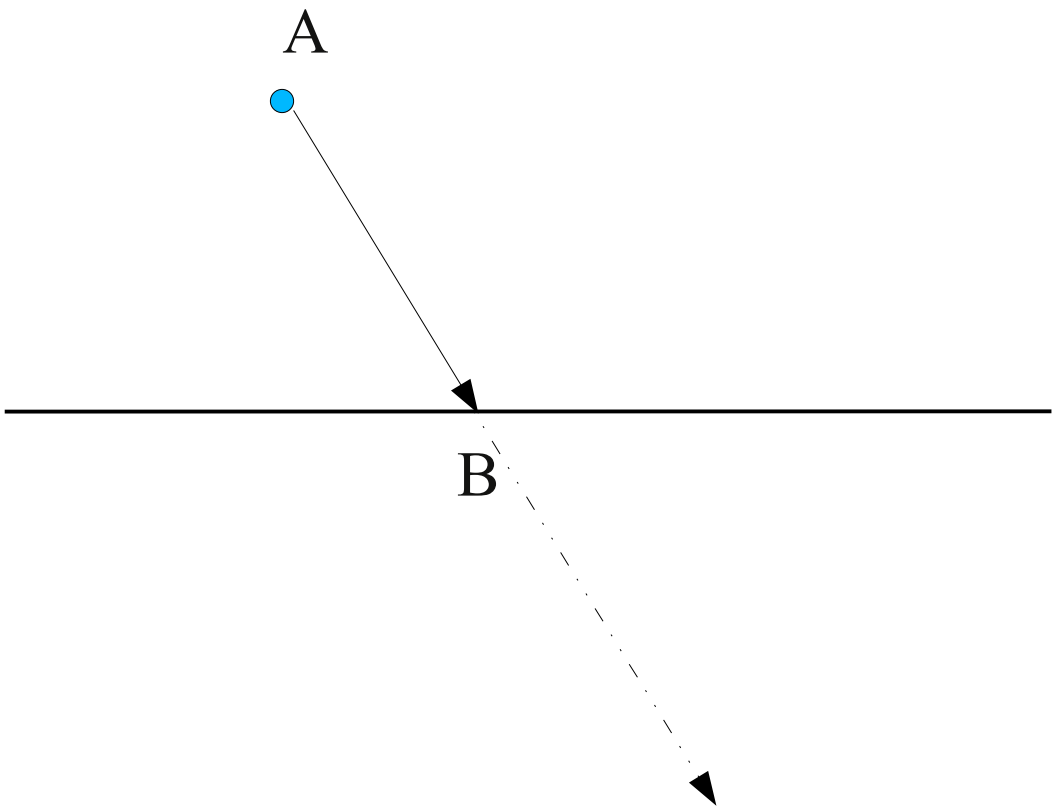
Idealizing assumptions for refraction

- The ball moves at a constant speed before contact and at a constant speed after contact
- The ball moves toward a linen sheet, which can be punctured
- Contact with the linen causes the ball to lose (case 1) or gain (case 3) some speed
- The size, shape, weight of ball have no effect on the motion of the ball

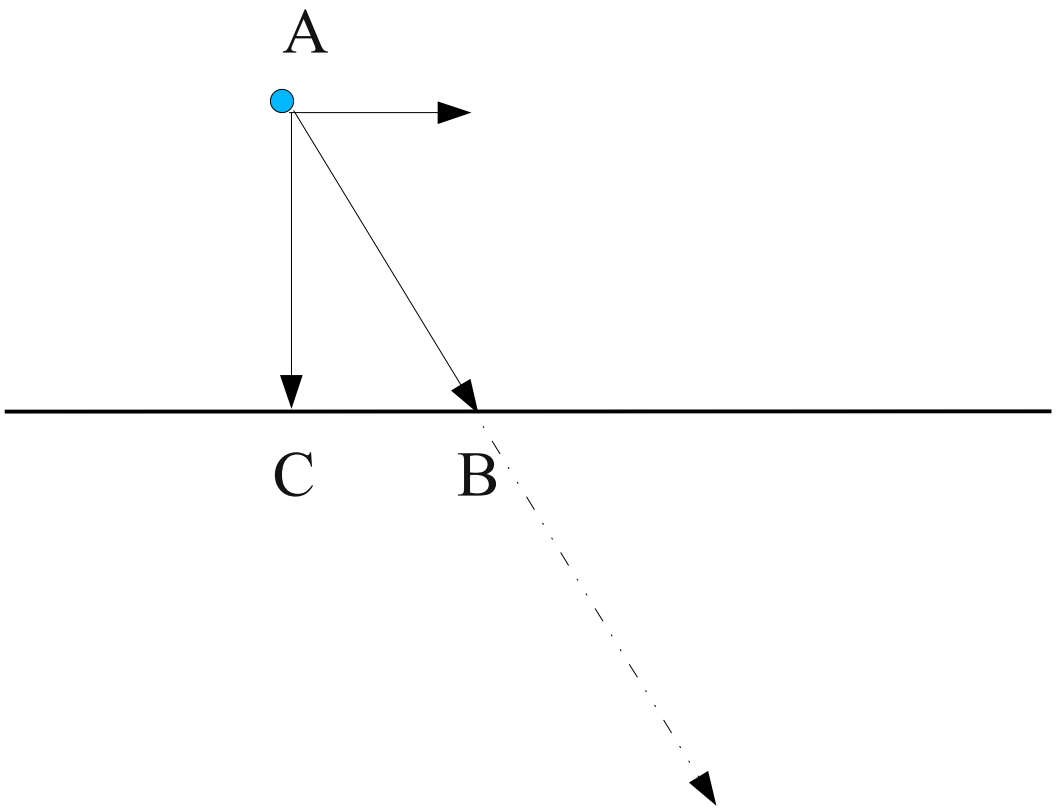
New theoretical assumptions for refraction

- The linen offers opposition to the ball in the downward direction
- The linen offers no opposition in the horizontal direction

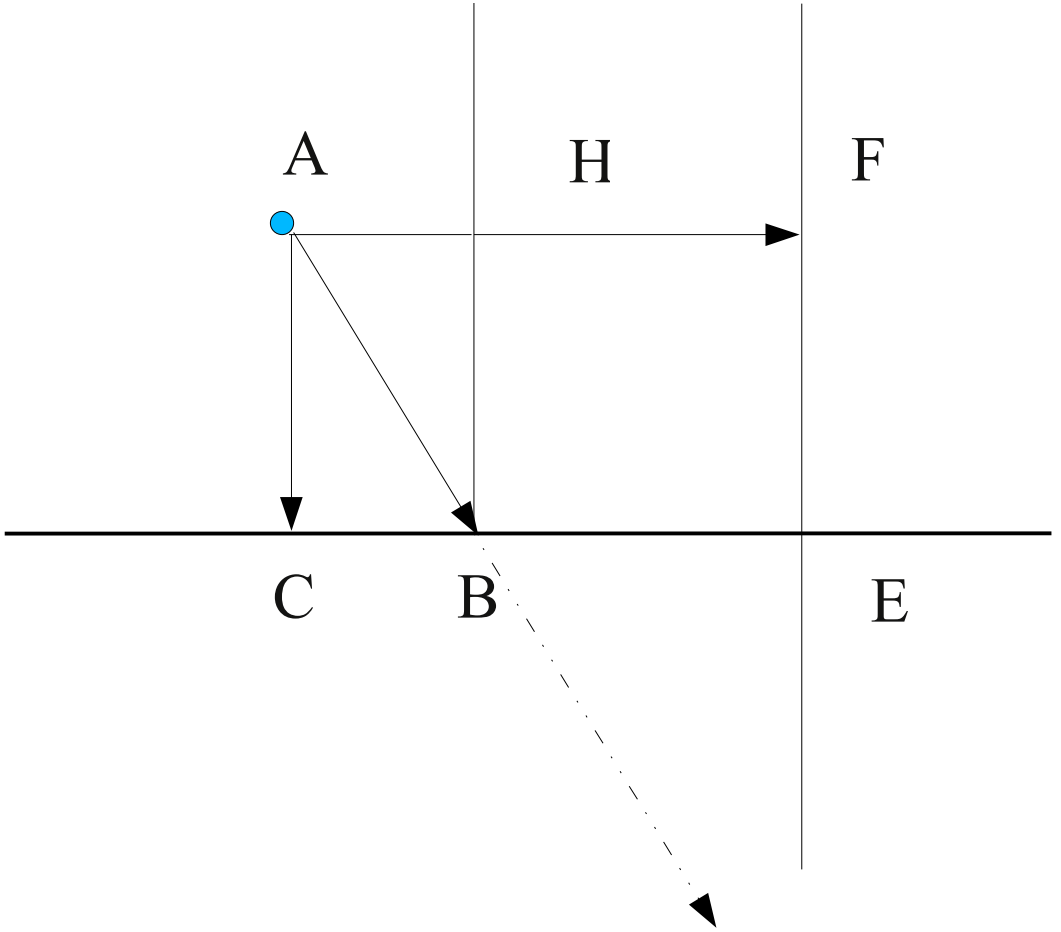
Case 1: Representation of the initial motion at a steep angle, with speed to decrease upon contact



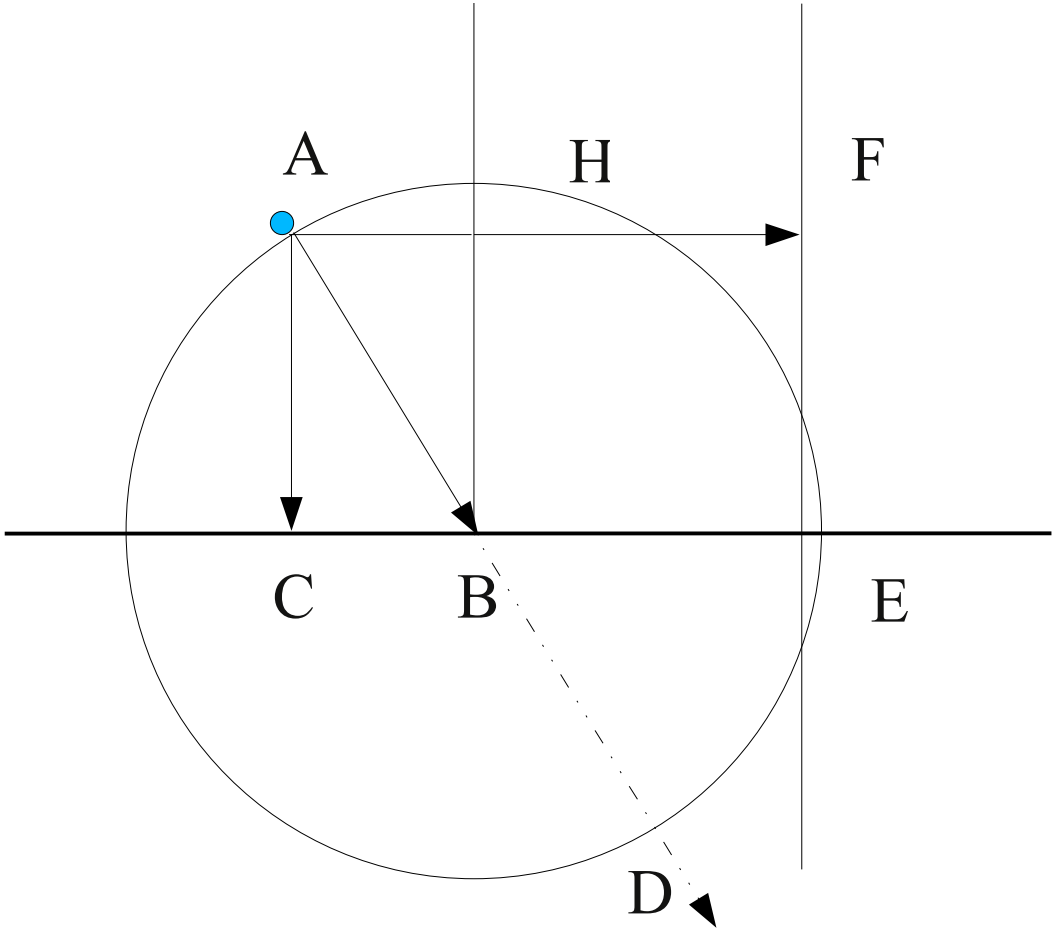
Decomposition of horizontal and vertical components



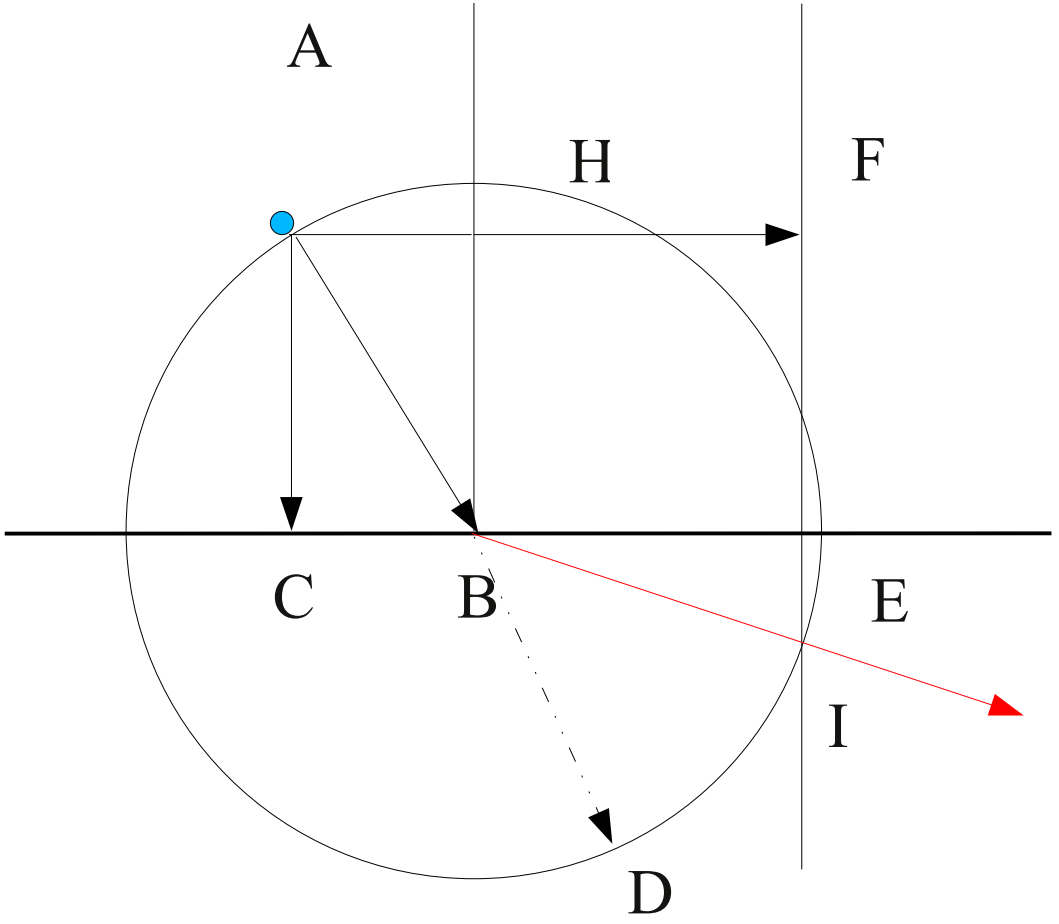
Doubling the size of the horizontal component



Duplication of the distance of the initial motion



Determination of the path after contact with the linen



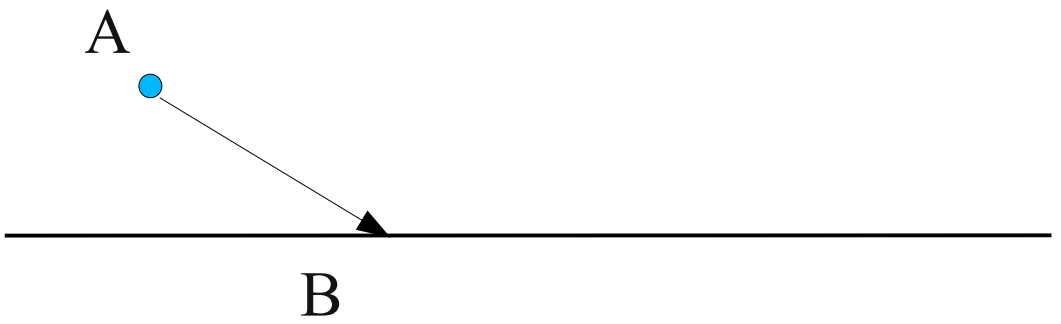
Descartes's reasoning

- The ball returns to the circumference of the circle from the point of contact B in twice the time it took to get from A to B, since it lost half its speed
- In twice the time, it covers twice the distance on the second horizontal component as it did on the first, since (by assumption) the horizontal determinant did not change
- So, it would have to arrive at line EF at the same time it arrives on the circumference: at point I

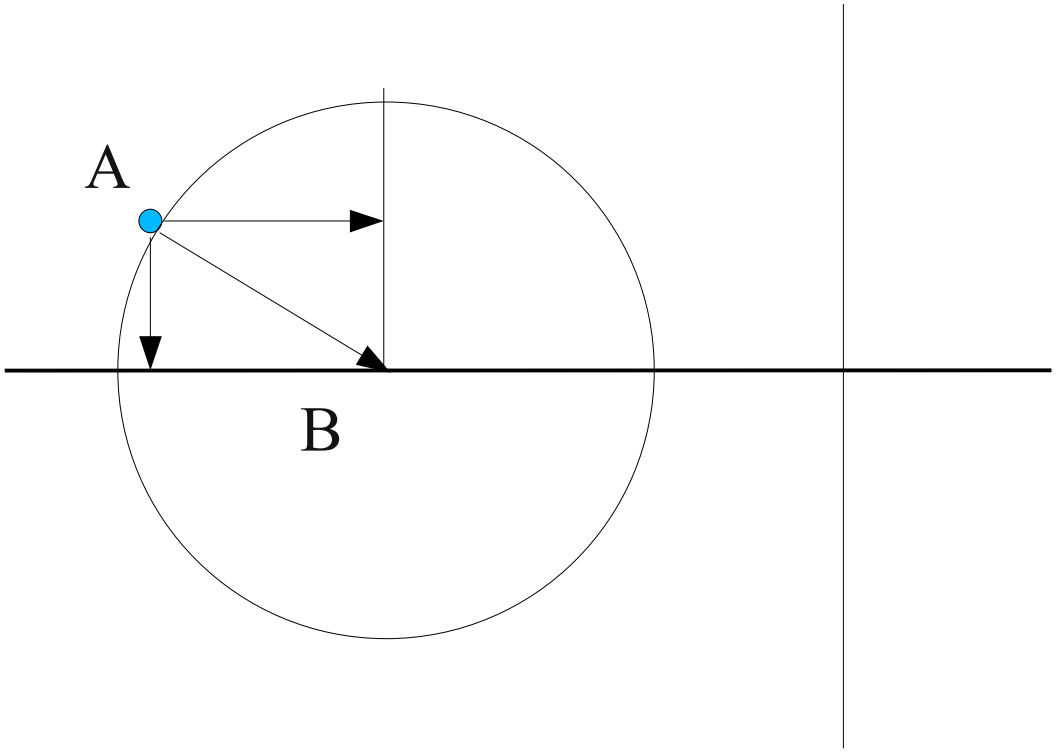
A more common case

- The ball strikes the surface of water and continues to move through the water
- On impact, the water reduces the speed of the ball by one-half, as before, but does not affect its horizontal determinant
- So the effect is the same
- Objection: the water would continue to reduce the speed of the ball
- Response: not given the idealizing assumptions

Case 2: Representation of the initial motion a shallow angle



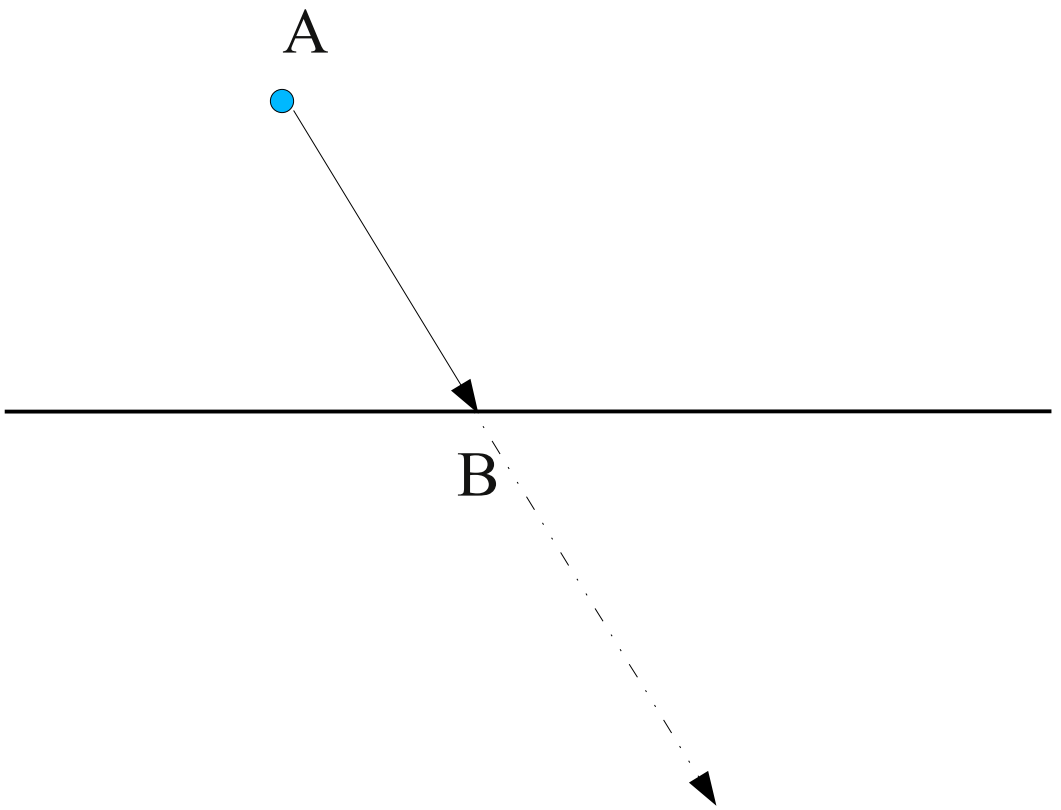
Doubled horizontal distance and identical composite distance



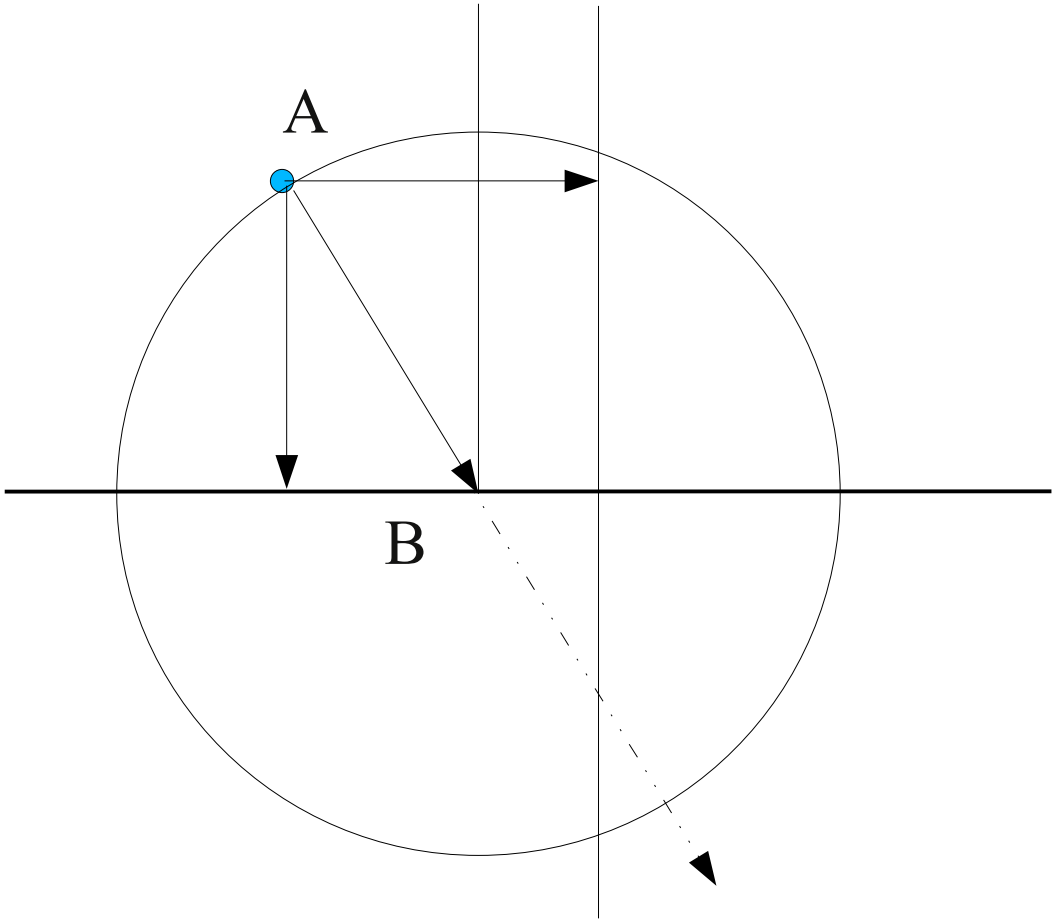
Reflection, not refraction

- When the ball enters at a steep angle, the vertical component is great enough it to allow completion of the horizontal component within the circle
- At the limit, the ball is dropped straight down, and there is no effect on the horizontal at all
- When the ball enters at a shallow angle, the vertical component is not great enough to compensate for the increased horizontal component
- So, the ball is reflected, as with skipping rocks

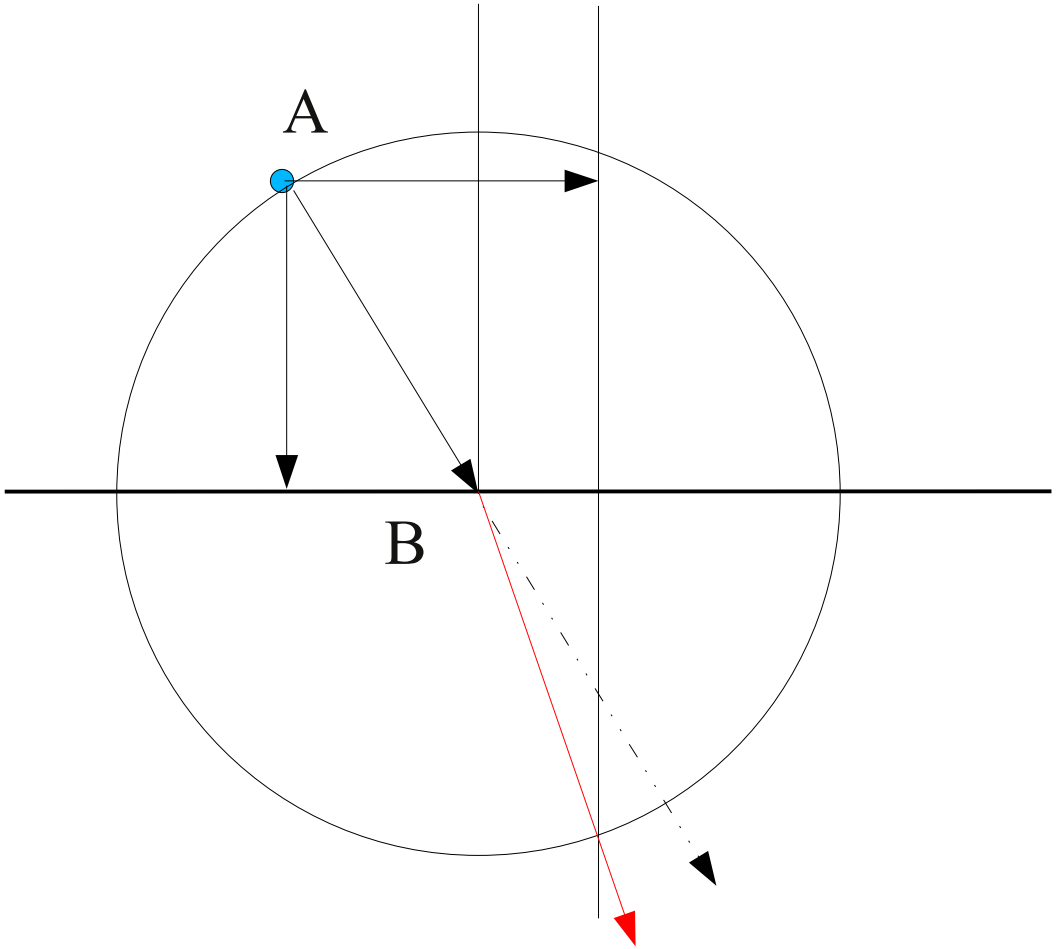
Case 3: Representation of the initial motion at a steep angle, with speed to increase upon contact



Halved horizontal distance and identical composite distance



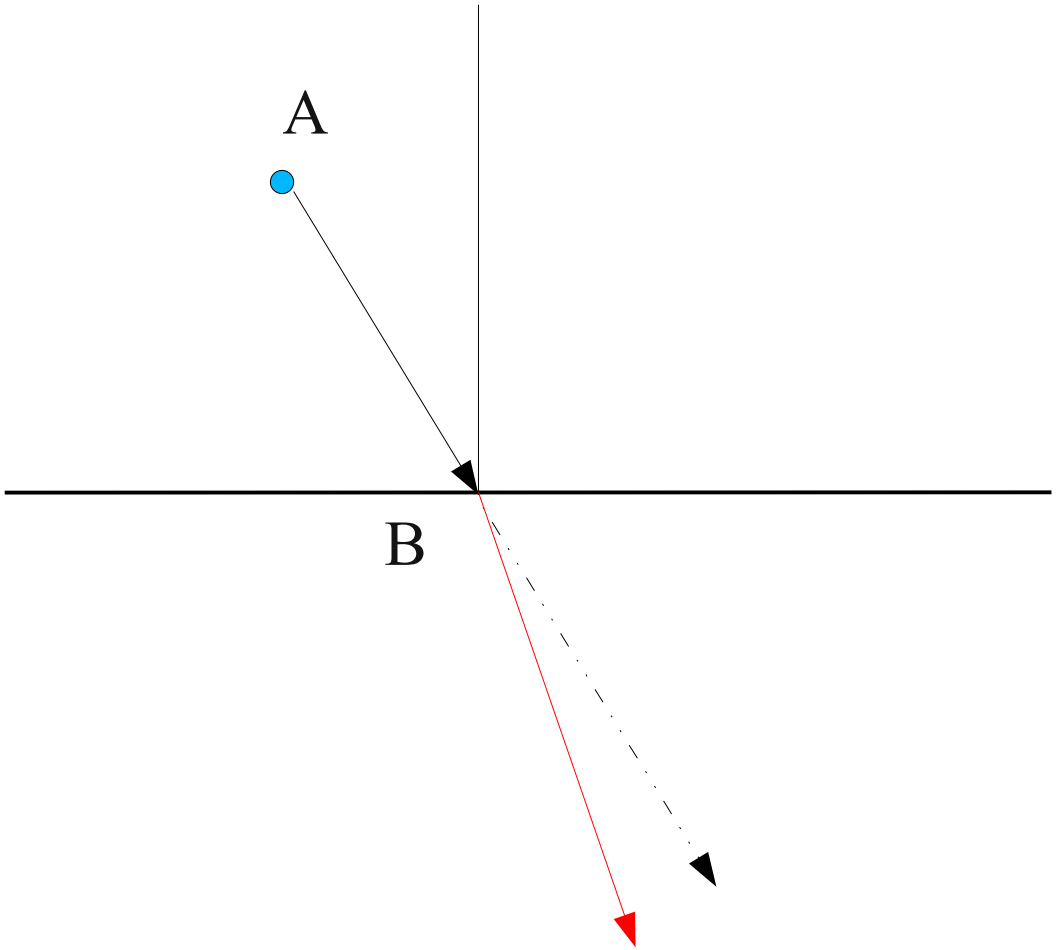
Determination of the path after contact with the linen



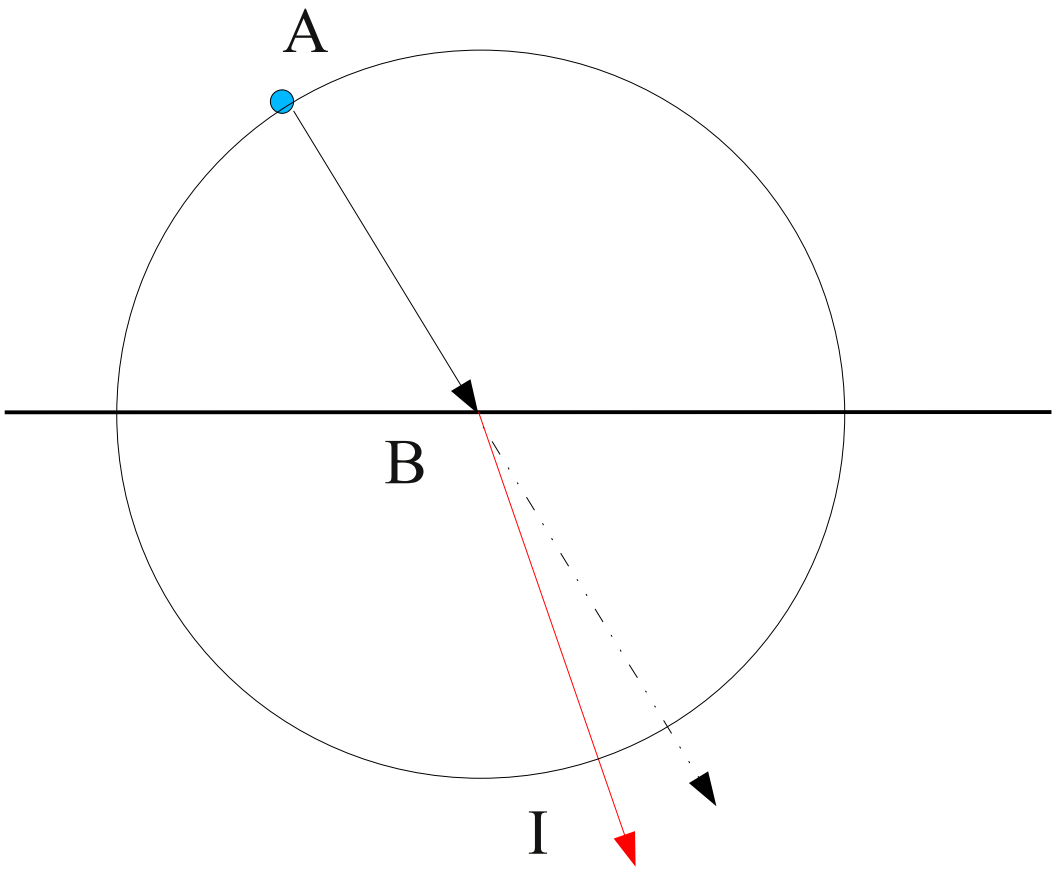
Reversing the model

- It has been established that degree to which the speed is increased or decreased by entry into the medium determines the path BI after contact
- Descartes asserts that if the path is determined by BI, then the change of speed can be calculated

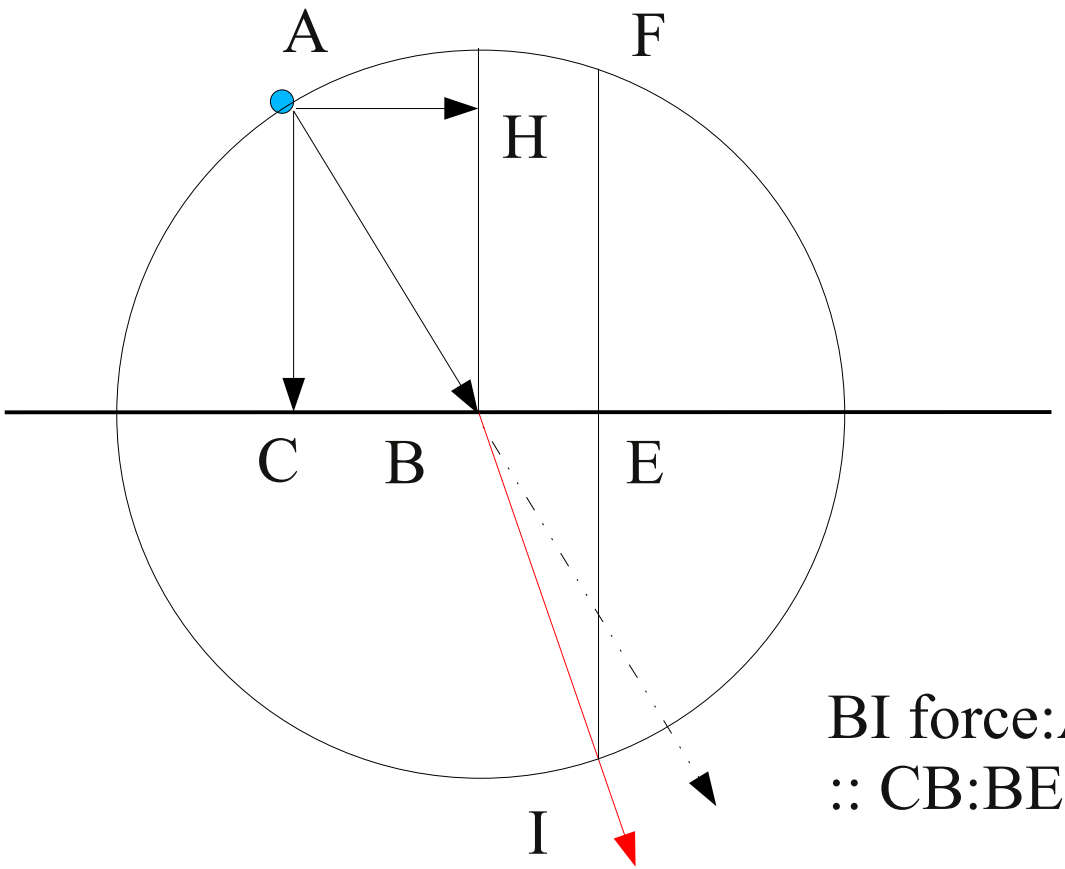
Path through the medium is given



Duplication of initial distance AB



Determination of relative speed (i.e., horizontal component)

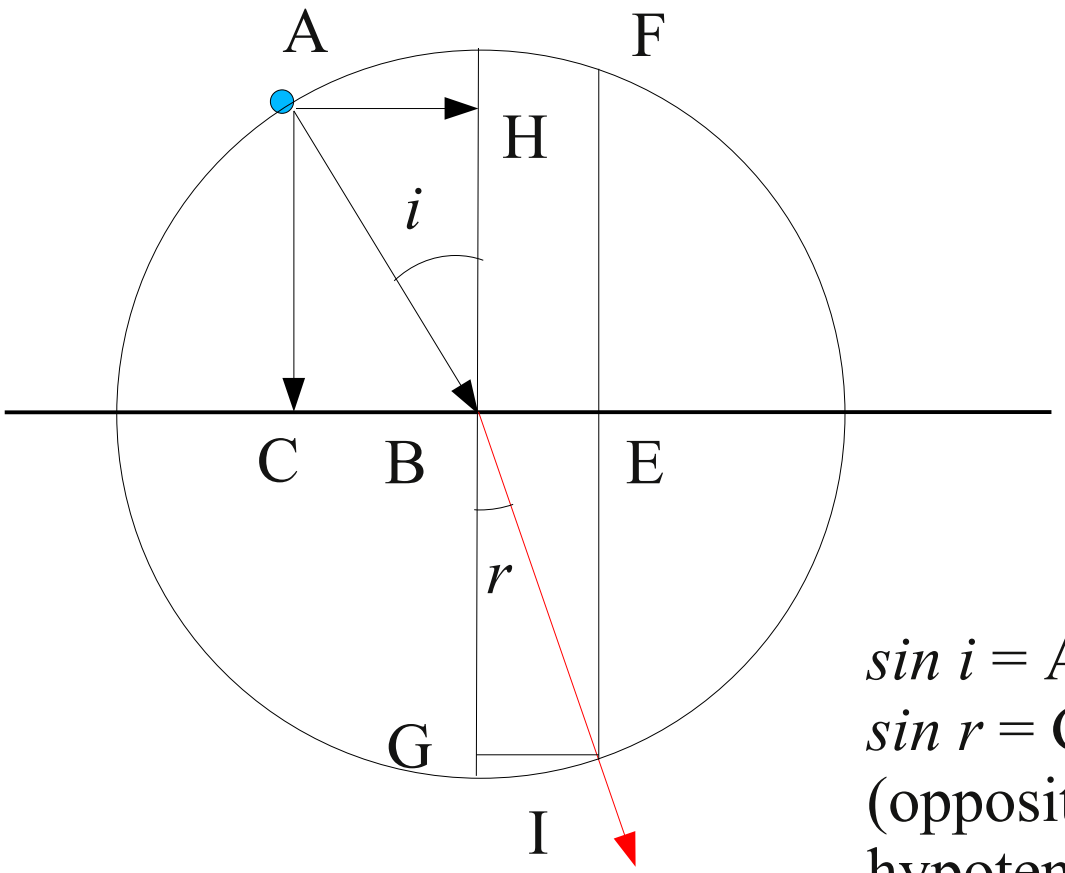


BI force:AB force
:: CB:BE

Conclusions

- These conclusions apply to light if light behaves the same way as these idealized bodies
- The angle formed with the surface and actual path varies depending on the difference in ease of penetration between the two media
- The angle will be less sharp on the side of the body that is more easily penetrated
- The degree of the angle varies exactly with the degree of the difference in degree of ease

Angles of incidence and refraction



sin i = AH/AB
sin r = GI/BI
(opposite side/
hypotenuse)

Proportionality of angles

- Differences between ease of penetration of the media are modeled by the lines CB and BE
- Therefore, the proportions of the angles vary with the proportions of the lines CB and BE
- If we take the proportions of the angles to be their sines, then they are proportional to CB and BE
- GI = BE, so GI/BI = BE/BI. AB = BI, so the denominators are canceled out in CB/AB and BE/BI
- *Sin r* : *sin i* :: CB : CE

Snell's Law

- Descartes's result is equivalent to Snell's law: $\sin i = n \sin r$, where n is a constant depicting the medium
- Snell's constant n is reflected in Descartes's proportions of speeds
- In Descartes's first example, CB = ½ BE
- So, CB/BH = ½ BE/BI, since BH = BI
- So, $\sin i = \frac{1}{2} \sin r$
- Descartes was not the first to discover this fact